# LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2025**

**PST 2503 – SAMPLING THEORY**

Date: 29-04-2025 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

## SECTION A

**Answer ANY FOUR of the following** **4 x 10 = 40 Marks**

1. Provide an example problem and prove that more than one unbiased estimator can exist for a given sampling design.

2. Examine whether $T_1(s) = \frac{1}{n(s)} \sum_{i \in s} Y_i$ and $T_2(s) = [\max_{i \in s}\{Y_i\} + \min_{i \in s}\{Y_i\}]/2$ are unbiased for $\bar{Y}$ under the sampling design, $P(s) = \begin{cases} \frac{1}{4} & if\ n(s) = 3 \\ 0 & otherwise \end{cases}$.
Given $Y_1 = 7$, $Y_2 = 3$, $Y_3 = 4$, and $Y_4 = 2$.

3. For any sampling design P(.), show that:
   a) $E_P[I_i(s)] = \pi_i\ ;\quad i = 1, 2, \ldots, N.$ (5)
   b) $E_P[I_i(s) I_j(s)] = \pi_{ij}\ ;\quad i, j = 1, 2, \ldots, N;\ i \neq j.$ (5)

4. Explain Random Group Method in detail. Also, prove that an unbiased estimator of Y under random group method is $\hat{Y}_{RG} = \sum_{i=1}^{n} \frac{y_i}{x_i} T_x(i)$.

5. For any fixed size sampling design P(.), prove that
   a. $\sum_{j=1}^{N} \pi_{ij} = (n-1)\,\pi_i\ ;\ i = 1, 2, \ldots, N;\ j \neq i$ (4)
   b. $\sum_{j=1}^{N} (\pi_i \pi_j - \pi_{ij}) = \pi_i (1 - \pi_i);\ i = 1, 2, \ldots, N;\ j \neq i$ (6)

6. Explain proportional allocation in Stratified Sampling and deduce $V(\hat{Y}_{St})$ under this allocation.

7. Obtain Hartley – Ross unbiased ratio type estimator for population total.

8. Show that $\widehat{Y_{LR}}$ is more efficient than $\widehat{Y_R}$ unless $\beta$ = R. Also prove that the ratio estimator is a particular case of regression estimator.

## SECTION B

**Answer ANY THREE of the following** **3 x 20 = 60 Marks**

9. (a) Prove that unbiasedness of an estimator depends on the sampling design. (10)

   (b) In SRSWOR, show that (10)
   (i) $\pi_i = \frac{n}{N}\ ;\ i = 1, 2, \ldots, N$ , and
   (ii) $\pi_{ij} = \frac{n(n-1)}{N(N-1)}\ ;\ i,\ j = 1, 2, \ldots, N\ \ and\ i \neq j.$

10. Explain in detail Warner's Model and find the estimated variance of $\hat{\Pi}_A$.

11. Describe Hansen-Hurwitz Estimator and show that $\hat{Y}_{HHE}$ is unbiased for Y. Also, show that $v(\hat{Y}_{HHE}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{Y}_{HHE}\right)^2$.

12. (a) Describe Simmon's Unrelated Randomized Response Model and estimate the population proportion $\Pi_A$ when $\Pi_Y$ is unknown. (12)

(b) Describe the linear regression estimation procedure for estimating the population total "Y". (8)

13. Under Midzuno sampling Design, show that

a. the first order inclusion probability is $\Pi_i = \frac{N-n}{N-1}.\frac{X_i}{X} + \frac{n-1}{N-1}$, i=1,2…, N. (8)

b. the second order inclusion probability is $\Pi_{ij} = \frac{(N-n)(n-1)}{(N-1)(N-2)}.\frac{X_i+X_j}{X} + \frac{(n-1)(n-2)}{(N-1)(N-2)}$, $i \neq j = 1, 2, \dots, N$. (12)

14. (a) Obtain the approximate bias and MSE of the Ratio Estimator. (8)

(b) Verify if the Hansen-Hurwitz estimator $\hat{Y}_{dhh}$ under double sampling is unbiased for Y and find $V(\hat{Y}_{dhh})$. (12)

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